symmetric tensor:

(i) Let us consider a contravariant tensor X^{ij} of rank 2 and if

$$X^{ij} = X^{ji}$$

then $X^{ij} \longrightarrow$ symmetric tensor

Similarly for X_{ij}, if $X_{ij} = X_{ji}$
$\longrightarrow X_{ij} \rightarrow$ Symmetric

(ii) Consider an arbitrary rank tensor, for example X^{ijk}_{lmn}, if

$$X^{ijk}_{lmn} = X^{jik}_{lmn}$$

then X^{ijk}_{lmn} is Symmetric in the first two contravariant indices.

Similarly if, $X^{ijk}_{lmn} = X^{ijk}_{nml}$, then

$X^{ijk}_{lmn} \longrightarrow$ symmetric in the first and third covariant indices.

Antisymmetric tensor:

(i) Consider the previous example. i.e., $X^{ij} \rightarrow$ tensor quantity and if

$$X^{ij} = -X^{ji}$$

$$\Rightarrow X^{ij} \text{ is antisymmetric}$$

Similarly for X_{ij}, if

$$X_{ij} = -X_{ji}$$

then $X_{ij} \rightarrow$ antisymmetric.

(ii) For tensor X^{ijk}_{lmn} if

$$X^{ijk}_{lmn} = -X^{jik}_{lmn}$$

$X^{ijk}_{lmn} \rightarrow$ antisymmetric in the first two contravariant indices.

Again if $X^{ijk}_{lmn} = -X^{ijk}_{nml}$, then

$X^{ijk}_{lmn} \rightarrow$ antisymmetric in the first and third covariant indices.

Example of symmetric tensor: —

(i) Kronecker delta $\rightarrow \delta_{ij}$ or δ^{ij} etc

(ii) For an isotropic system, stress tensor σ_{ij} or σ^{ij} is symmetric tensor.